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Paper, 'Experiments to determine the density of the earth' by Henry Cavendish

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68 Experiments to determine the density of the earth. By Henry Cavendish Esq. F.R.S. and A.S.
Read June 21. 1798

Many years ago the late Revnd John Michell of this society contrived a method of deter:mining the density of the earth by rendering sensible the attraction of small quantities of matter; but as he was engaged in other pursuits, he did not complete the apparatus till a short time before his death, & did not live to make any experiments with it. After his death the apparatus came to the Revnd ~~Francis~~ John Hyde Wollaston Jacksonian Professor at Cambridge who not having conveniences for making experiments with it in the manner he could wish was so good as to give it to me

The apparatus is very simple it consists of a wooden arm 6 feet long made so as to unite great strength with little weight

This arm is suspended in an horizontal position by a slender wire 40 inches long & ~~from~~ to each extremity is ~~suspended~~ hung a leaden ball about 2 inches in diameter & the whole is inclosed in a narrow wooden case to defend it from the wind

As no more force is required to make this arm turn round on its center than what is necessary to twist the suspending wire it is plain that if the wire is sufficiently slender the most minute force such as the attraction of a leaden weight a few inches in diameter will be sufficient to draw the arm sensibly aside The weights which Mr Michell intended to use were 8 inches diam one of these was to be placed on one side the case opposite to one of the balls & as near it as could conveniently be done & the other on the other side opposite to the other ball so that the attraction of both these weights would conspire in drawing the arm aside

& when its position as affected by these weights was ascertaind the weights were ~~intended~~ to be removed to the other side of the case so as to draw the arm the contrary way & the position of the arm was to be again determined & consequently ½ the difference of these positions would ~~shew~~ shew how much the arm was drawn aside by the attraction of the weights

In order to determine from hence the density of the ⊕ it is necessary to ascertain what force is required to draw the arm aside ~~by~~ through a given space this Mr Michell intended to do by putting the arm in motion & observing the time of its vibrations from which it may easily be computed *

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* Mr Coulomb has in a variety of cases used a contrivance of this kind for trying small attractions but Mr Michell informed me of his intention of ~~trying~~ making this experiment & of the method he intended to use before the publication of any of Mr Coulombs experiments

Mr Michell had prepared 2 wooden stands on which the leaden weights were to be supported & pushed forwards till they came

~~Mr Michell had prepared a wooden stands to support the leaden weights & to slide them till they came~~ almost in contact ~~with the~~ with the case but he seems to have intended to move them by hand

As the force with which these ball are ~~wouldn't be~~ attracted by these weights is excessively minute not more than $\frac{1}{50.000.000}$ of their weight it is plain that a very minute disturbing force will be sufficient to destroy the success of the experiment & from the following experiments it will appear that the disturbing force most difficult to guard against is that arising from the variations of heat & cold for if one side of the case is warmer than the other the air in contact with it will be rarified & in consequence will ascend while that on the other side will descend & produce ~~which will cause~~ a current which will draw the arm sensibly aside *

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* Mr Cassini in observing the variation compass placed by him in the observatory ~~of~~ which

observe the motion of the arm from without.
by means of a telescope & to suspend the leaden
weights in such manner that I could move
them without entering into the room. This
~~alteration~~ difference in the manner of observing ~~made~~ rendered
it necessary to make some alteration in Mr
Michells apparatus & as there were some parts
of it which I thought not so convenient as I
could be wished I chose to make the greatest part
of it afresh

Fig. 1 is a longitudinal vertical section
through the instrument & the building in
which it is placed ABCDDCBAEFFE is
the case x & x are the 2 balls which are
suspended by the wires hx from the arm
ghmh which is itself suspended by the slender
wire gl. This arm consists of a slender deal
rod hmh strengthend by a silver wire hgh
by which means it is made strong enough
to support the balls, though very light*

* Mr Michells rod was intirely of wood & was

As I was convinced of the necessity of guarding against this source of error I resolved to place the apparatus in a room which should remain constantly shut & to

was constructed so as make very minute changes of position visible ~~&~~ in which the needle was suspended by a silk thread found that standing near the box in order to observe drew the needle sensibly aside which I have no doubt was caused by this current of air It must be observed that ~~the case~~ his compass box was of metal which transmits heat faster than wood & also was many inches deep both which causes served to increase the current of air To diminish the effect of this current it is by all means advisable to make the box in which the needle plays not much deeper than ~~what~~ is necessary to prevent the needle from striking against the top & bottom

The case is supported & set horizontal by 4 screws resting on posts ~~set~~ fixed firmly into the ground 2 of them are represented in the figure by S & S the 2 others are not represented to avoid confusion GG are the end walls of the building W & W are the leaden weights which are suspended by the copper rods RrPrR & the wooden bar Rr from the center pin Pp This pin passes through a hole in the beam HH perpendicularly over the center of the instrument & turns round in it being ~~being~~ prevented from falling by the plate

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much stronger & stiffer than this though not much heavier but as it had warped when it came to me I chose to make another & ~~made it in~~ preferred this form partly as being easier to construct & meeting with less resistance from ~~made & being less resisted~~ by the air & partly because from its being of a less complicated form ~~it was easier to~~ I could more easily compute how much it was attracted by the weights p MM is a pulley fastend to this pin & Mm a cord wound round the pulley ~~it~~ & passing through the end wall by which the observer may turn it round & thereby move the weights from one situation to the other

Fig. 2 is a plan of the instrument AAAA the case SSSS the 4 screws for supporting it hh the arm & balls W & W the weights MM the pulley for moving them When the weights are in this position both conspire in drawing the arm in the direction hW but when they are removed to the situation w & w represented by the dotted lines both conspire in drawing the arm in the contrary direction hw These weights are ~~were~~ prevented from striking the instrument by pieces of wood which stop them as soon as they come within $\frac{1}{5}$ of an inch of the case the pieces of wood ~~were~~ fastend to the wall of the building & it

I find ~~was found~~ that the weights ~~might~~ may strike against them with considerable force without sensibly shaking the instrument

In order to determine the situation of the arm slips of ivory are placed within the case as near to each end of the arm as can be done without danger of touching it & are divided to 20ths of an inch Another small slip of ivory is placed at each end of the arm serving as a Vernier & subdividing these divisions into 5 parts so that the position of the arm may be observed with ease to 100ths of an inch & may be estimated to less These divisions are viewed by means of the short telescopes T & T (fig 1) through slits cut in the end of the case & stopt with glass They are enlightened by the lamps L & L with convex glasses placed so as to throw the light on the divisions no other light being admitted into the room

The divisions on the slips of ivory run in the direction Ww (fig 2) so that when the weights are placed in the positions w α w represented by the dotted circles the arm is drawn aside in such ~~manner~~ direction as to make the index point to a higher number on the slips of ivory for which reason I call this the positive position of the weights

K (fig 1) is a wooden rod which by means of an endless screw turns round the support to which the wire gl is fastend & thereby enables the observer to turn round the wire till the arm settles in the middle of the case without danger of touching either side The wire gl is fastend to ~~the plate h~~ its support at top & to the center of the arm at bottom by brass clips in which it is pinched by screws

In these 2 figures the different parts are drawn nearly in the proper proportion to each other & on a scale of one to 10

Before I proceed to the account of the experiments it will be proper to say something of the manner of observing. Suppose the arm to be ~~at~~ at rest & its position to be observed ~~[illegible]~~ the weights be then moved the arm will not only be drawn aside thereby but it will be made to vibrate & its vibrations will continue a great while so that in order to determine how much the arm is drawn aside it is necessary to observe the extreme points of the vibrations & from thence to determine the point which it would rest at if its motion was destroyed or the point of rest as I shall call it. To do this I observe 3 successive extreme points of a vibration & take the mean between the first & 3rd of these points as the extreme point of vibration in one direction & then assume the mean between this & the 2nd extreme as the point of rest for as the vibrations are continually dimini: :shing it is evident that the mean

The next thing to be determind is the time of vibration which I find in this manner I observe the 2 extreme points of a vibration & also the times at which the arm arrives at 2 given divisions between these extremes taking care as well as I can guess that these divisions shall be on different sides of the middle point ~~of the~~ & not very far from it I then compute the middle point of the vibration & by proportion find the time at which the arm comes to this middle point I then after a number of vibrations repeat this operation & divide the interval of time between the coming of the arm to these 2 middle points by the number of vibrations which gives the time of one vibration The following example will explain what is here said more clearly between 2 extreme points will not give the true point of rest

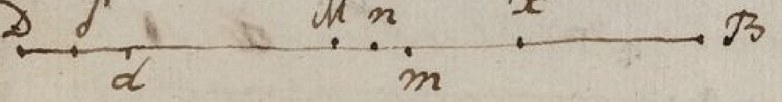
It may be thought more exact to observe many extreme points of vibration so as to find the point of rest by different sets of 3 extremes & to take the mean result but it must be observed that notwithstanding the pains taken to prevent any disturbing force the arm will seldom remain perfectly at rest for ~~even~~ an hour together for which reason it is best to determine the point of rest from observations made as soon after the motion of the weights as possible

~~Another thing to be determined is the time of a vibration but before I mention my manner of doing this it will be proper to examine how much the time of the whole or any part of a vibration is affected by the resistance of the air~~

extreme points	division	time	point of rest	time mid vib
27.2		h ′ ″		
	25	10.23.4	— —	10.23.23
	24	57		
22.1	—	—	24.6	
27	—	—	24.67	
22.6	—	—	24.75	
26.8	—	—	24.8	
~~[illegible]~~ 23	~~[illegible]~~	~~11.5.22~~	24.85	
26.6	~~24~~	~~6.48~~	24.9	
~~23.4~~	25	11.5.22	— —	11.5.22
	24	6.48		
23.4				

The first column contains the extreme points of the vibrations the 2nd the intermediate divisions & the 3rd the time at which the arm came to those divisions & the 4th the point of rest which is thus found The mean between the first & 3rd extreme point is 27.1 & the mean between this & the 2nd extreme point is 24.6 which is the point of rest as found by the 3 first extremes In like manner the point of rest found by the 2nd 3rd & 4th extreme is 24.7 & so on The 5th column is the time at which the arm came to the middle point of the vibration which is thus found The mean between 27.2 & 22.1 is 24.65 & is the middle point of the first vibration & as the arm came to 25 at $10^h.23'.4''$ & to 24 at 10.23.57 we find by proportion that it came to 24,65 at 10.23.23 In like manner the arm came to the middle of the 7^{th} vibration at 11.5.22 & therefore 6 vibrations were performed in 41.59 or one vibration in $7'.6$

To judge of the propriety of this method we must consider in what manner the vibr is affected by the resistance of the air & by the motion of the point of rest



Let the arm during the first vibration move from D to B & during the 2^{nd} from B to d Bd being less than DB on account of the resistance Bisect DB in M & Bd in m & bisect Mm in n & let x be any point in the vibration Then if the resistance is propor:tional to the square of the velocity ~~& if T is taken to the time of one vibration as the~~

~~diameter of a circle to its semicircumference~~
the whole time of a vibration is very little alterd but if T is taken to the time of one vibration as the diameter of a circle to its semicircumference the time of moving from B to n exceeds $\frac{1}{2}$ a vibration by $\frac{T \times Dd}{8\,Bn}$ nearly & the time of moving from B to m falls short of $\frac{1}{2}$ a vibration by as much & the time of moving from B to x in the 2nd vibration exceeds that of moving from x to B in the first by $\frac{T \times Dd \times Bx^2}{4Bn^2 \times \sqrt{Bx \times x\delta}}$ supposing Dd to be bisected in δ so that if a mean is taken between the time of the first arrival of the arm at x & its returning ~~to~~ back to the same point this mean will be earlier than the true time of its coming to B by $\frac{T \times Dd \times Bx^2}{8Bn^2 \sqrt{Bx \times x\delta}}$

The effect of motion in the point of rest is that when the arm is moving in the same direction as the point of rest the time of moving from one extreme point of vibration to the other is increased & it is diminished when they are moving in contrary directions but if the point of rest moves uniformly the time of moving from one extreme to the middle point of the vibration will be equal to that of moving from the middle point to the other extreme & moreover the time of 2 successive vibrations will be very little alterd & therefore the time of moving from the middle point of one vibration to the middle point of the next will also be very little alterd

It appears therefore that on account of the resistance of the air the time at which the arm comes to the middle point of the vibration is not exactly the mean between ~~the~~ times of its coming to the extreme points which causes some inaccuracy in my method of finding the time of a vibration It must be observed however that as the time of coming to the middle point is before the middle of the vibration both in the first & last vibration & in general is nearly equally so the error produced from this cause must be inconsiderable & on the whole I see no method of finding the time of a vibration which is liable to less objection

The time of a vibration may be determined either by previous trials or it may be done at each experiment by ~~determining~~ ascertaining the time of the vibrations which the arm is actually put into by the motion of the weights but there is one advantage in the latter method namely that if there should be any acciden: :tal attraction such as electricity in the glass plates through which the motion of the arm is seen which should increase the force necessary to draw the arm aside it would also diminish the time of vibration & consequently the error in the result would be much less when the force required to draw the arm aside was deduced from experiments made at the time than ~~from previous experiments~~ when it was taken from previous experiments -

Account of the experiments

In my first experiments the wire by which the arm was suspended was $39\frac{1}{4}$ inches long & was of copper silverd one foot of which weighd $2\frac{4}{10}$ grains Its stiffness was such as to make the arm vibrate in about 15 minutes I immediately found ~~indeed~~ that it was not stiff enough as the attraction of the weights drew the balls so much aside as to make them touch the sides of the case I however chose to make some experiments with it before I changed it

In this trial the rods by which the leaden weights were suspended were of iron for as I had taken care that there should be nothing magnetical in the arm it seemed of no signification whether the rods were magnetical or not but for greater security I took off the leaden weights & tried what effect the rods would have by themselves Now I find by computation that the attraction of gravity of these rods on the balls is to that of the weights nearly as 17 to 2500 so that as as the attraction of the weights appeard by the foregoing trial to be sufficient to draw the arm aside by about 15 divisions the attraction of the rods alone should draw it aside about $\frac{1}{10}$ of a division & therefore the motion of the rods from one near position to the other should move it about $\frac{1}{5}$ of a division

The result of the experiment was that for the first 15 minutes after the rods were moved from one near position to the other very little motion was produced in the arm & hardly more than ought to be produced by the action of gravity but the motion then increased so that in about $\frac{1}{4}$ or $\frac{1}{2}$ an hour more it was found to have moved $\frac{1}{2}$ or $1\frac{1}{2}$ divisions in the same direction that it ought to have done by the action of gravity On returning the irons back to their former position the arm moved backwards in the same manner that it before moved forward

It must be observed that the motion of

the arm in these experiments was hardly more than would sometimes take place without any apparent cause but yet as in 3 experiments which were made with these rods the motion was constantly of the same kind though differing in quantity from $\frac{1}{2}$ to $1\frac{1}{2}$ of a division there seems great reason to think that it was produced by the rods.

As this effect seemed to me to be owing to magnetism though it was not such as I should have expected from that cause I changed the iron rods for copper & tried them as before the result was that there still seemed to be some effect of the same kind but more irregular so that I attributed it to some accidental cause & therefore hung on the leaden weights & proceeded with the experiments.

It must be observed that the effect which seemed to be produced by moving the iron rods from one near position to the other was at a medium not more than ~~one~~ 3/4 of division whereas the effect produced by moving the weight from the midway to the near position was about 15 divisions so that if I had continued to use the iron rods the error in the result caused thereby could hardly have exceeded $\frac{1}{30}$ of the whole

Exp. 5

On Aug 5 the weights being in the midway position the arm was found stationary at 11.5 At 10.5 it moved weights to positive position

extreme points	division	Time	point of rest	time of middle of vibr
23,4				
27,6	24 –	39.20	25,82	
	25 – –			
24.7			26,07	
27,3			26,1	
25,1				

At 11.36 AM the weights were moved back to the midway position

5,		h ′ ″	
	11 – –	0.0.48	
	12 – –	1.30	
18,2			
6,6			
16,3			

Exp. 1 Aug 5

extreme points	divis	Time h ' "	point of rest	time of mid vib	
	11.4	9.42			
	11.5	55			
	11.5	10.5	11.5	weights in midway positi	

At 10'.5 weights moved to positive position

23.4					
27.6	—	— —	25.82		
24.7	—	— —	26.07		
27.3	—	— —	26.1		
25.1					

At 11.6 weights returned back to midway position

5.					
	11	0.0.48	— — —	0.1.13	
	12	1.30			
18.2	—	—	12		14.56
	12	16.29	— —	16.9	
	11	17.20			
6.6	—	—	11.92		14.36
	11	30.24	— —	30.45	
	12	31.11			
16.3	—	—	11.72		15.2
	12	45.58	— —	45.58	
	11	47.4			
7.7					

mean

Motion on moving from midway to pos. = 14.32

pos. to midway = 14.1

Time one vibr ——— 14'.53"

Exp. 1 Aug 5

extreme points	divis	Time	point of rest	Time of mid. vib.
	11.4	9.42. At		
	11.5	55		
	11.5	10.58	11.5	weights in midway position

moved weights to the positive position

extreme points	divis	Time	point of rest	Time of mid. vib.
23.4	- -			
27.6	~~24~~	- - - 20	25.82	
24.7	— 25	— —	26.07	
27.3	—	— —	26.1	
25.1				

At 11.36 moved weights to midway position

extreme points	divis	Time	point of rest	Time of mid. vib.
5				
	11	0.0.48	- - -	0.1.13
	12	1.30		
18.2	- -	- -	12	
6.6	-	- -	11.92	
16.3	—	— —	11.72	
	12 -	45.58		
	11	47.4	- - -	0.45.
7.7				

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It must be observed that in this experiment the attraction of the weights drew the arm from 11.5 to 25.8 so that if no contrivance had been used to prevent it the momentum acquired thereby would have carried it to near 40 & would therefore have made the balls to strike against the case To prevent this after the arm had moved near 15 divisions I returned the weights to the midway position & let them remain there till the arm came nearly to the extent of its vibration & then again moved them to the positive position whereby the vibrations were so much diminished that the ~~a~~ balls did not touch the sides & it was this which prevented my observing the first extremity of the vibration A like method was used when the weights were returned to the midway position & in the 2 following experiments

The vibrations in moving the weights from the midway to the positive position were so small that it was thought not worth while the time of its coming to the middle ~~of~~ point of only the first & last vibration to observe the time of the vibration When the weights were returned to the midway position I determined the time of the arm coming to the middle point of each vibration in order to see how nearly the times of the different vibrations agreed together In great part of the following experiments I contented myself with observing ~~the extreme vibrations~~ ~~[illegible]~~

extreme point	divis	time	point of rest	time of mid vib
	Exp. 2 Aug 6			
	11.	10.40.0		
	11 –	11		
	11 –	17		weights in midway position
	11	25	11.	
	weights moved to positive position			
29.3				
24.1	– –	–	– – – 26.87	
30	–	–	– 27.57	
26.2	– –	–	– 28.02	
29.7	–	–	– 28.12	
26.9	–	–	– 28.05	
28.7	–	–	– 27.85	
28.4	– –	–	– – 27.87	
28.4	weights returned to midway position			
6.	12			
	12	1.3.50		
	13	4.34	– –	1.4.1

extr points	divis	time	point rest	time mid vib			
18.5	12	1.18.29	12.37	1.18.53	14.52	1.18.53	14.52
	12	19.18	—				
6.5	12	—	11.67		15.48		14.46
	12	33.48	—	1.33.39	—	1.33.39	
	12	34.51	11		13.46		
15.2							13.46
—	13	45.8	—	1.47.25	—	1.47.25	
	12	46.22					
7.1					16.04		15.25
	11	2.3.48	10.75	2.3.26	—	2.2.50	
	12	5.18					
13.6							

Motion of arm on moving weights from midway to pos. = 15.87
pos. to mid = 15.45
Time of vibration = ~~15.4~~ = 14.42

Exp. 3 Aug 7 the weights being in the positive position & the arm a little in motion

31.5				
29	—	—	30.12	
31	—	—	30.02	
29.1				

Weights moved to midway

9					
	14	34.18	—	34.55	
	15	35.8			
20.5	—	—	14.8		14.44
	15	49.31	—	49.39	
	14	50.27			

extr.	divi	time	point rest	time mid	
9.2	—	— —	14.07		14.38
	14	11.5.7	— —	11.4.17	
	15	6.18			
17.4	—	— —	13.52		14.47
	14	18.46	— —	19.4	
	13	19.58			
10.1	—	— —	13.3		14.27
	13	33.46	— — —	33.31	
	14	35.26			
15.6					

Weights moved to the positive position

32				
	28	0.2.48	— —	0.2.59
	27	3.56		
23.7	—	— —	27.8	
31.8	—	— —	28.27	
25.8	—	— —	28.62	
	27	44.58		
	28	46.50	— —	0.47.40
31.1				

Mot arm on moving weights from pos. to mid = 15.22
mid to pos — 14.5

Time one vib when in mid point — 14.39
— — pos. point — 14.53

These experiments are sufficient to shew that the attraction of the weights on the balls is very

sensible & are also sufficiently regular to deter:
:mine the quantity of this attraction pretty nearly
as the extreme results do not differ from each
other by more than $\frac{1}{10}$ part but there is a
circumstance in them ~~which~~ the reason of which
does not readily appear namely that the
effect of the attraction seems to increase for
$\frac{1}{2}$ an hour or an hour after the motion of the
weights as it may be observed that in all
3 experiments the mean position kept increasing
for that time after moving the weights to the positive
position & kept decreasing after moving them
from the positive to the midway position

The first cause which occurred to me was
that possibly there might be a want of
elasticity either in the suspending wire or
something it was fastend to which might make
it yield more to a given pressure after a long
continuance of that pressure than it did at first
To put this to the trial I moved the index
so much that the arm if not prevented

on the sides of the case would have stood at about 50 divisions so that as it could not move farther than to 35 divisions it was kept in a position 15 divisions distant from that which it would naturally have assumed from the stiffness of the wire or in other words the wire was twisted 15 divisions After having remaind 2 or 3 hours in this position the index was moved back so as to leave the arm at liberty to assume its natural position

It must be observed that if a wire is twisted only a little more than its elasticity admits of that then instead of setting as it is called or acquiring a permanent twist all at once it sets gradually & ~~that~~ when it is left at liberty it gradually loses part of that set which it acquired so that if in this experiment the wire by having been kept twisted for 2 or 3 h. had gradually yielded to this pressure or had begun to set it would ~~when left at liberty~~ gradually restore it self when left at liberty & the point of rest would gradually move backwards but though the experiment was 2^d repeated I could not perceive any such effect

The arm was next suspended by a stiffer wire

Exp. 4 Aug 12

extr point	div	Time	point rest	time mid. vib	
	21.6	9.30			
	21.5	52			
	21.5	10.13			

Weights moved from midway to positive position

27.2					
22.1	~~25~~	~~10.23.4~~	24.6		
27	24	57	24.67		
27.6			24.75		
26.8			24.8		
26.9			24.85		
26.6			24.9		
26.4					
~~[illegible]~~					

Weights moved to the negative position

15					
~~12~~	17	19.25	—	10.20.31	
	19	20.41			
22.4	—	—	18.72		7.0
	20	26.45	—	27.31	
	19	27.22			
15.1	—	—	18.52		6.57
	19	35.28	—	34.28	
	20	48			

exts	div	time	point rest	time mid vib.	
21.5	–	— —	18.35		7.23
	20	10.40.23	— —	10.41.51	
	19	41.18			6.48
15.3	–	– – –	18.22		
	18	48.36	– – –	48.39	
	19	49.24			6.58
20.8	–	– –	18.1		
	19	54.45	– – –	55.37	
	18	55.45			
15.5					

Weights moved to positive position

31.3					
	25	11.10.25	— —	11.10.40	
	23	11.3			7.3
17.1	–	– –	24.02		
	22	17.6	— —	17.43	
	23	– 26			7.1
30.6	—	— —	24.17		
	25	24.33	— —	24.44	
	23	25.17			7.15
18.4	—	— —	24.32		
	23	31.21	— —	31.49	6.59
	25	32.9			
29.9	—	—	24.4		
	25	38.39	— —	38.48	
	23	39.31			7.16
19.4	—	— —	24.5		
	23	45.16	— —	45.54	
	25	46.12			
29.3					

Motion of arm on moving weights from mid neg to pos — 3.1
pos to neg — 6.18
neg to pos — 5.92

Time one vibration in neg position = 7.1
pos. position = 7.3

Exp. 5 Aug 20.
The weights being in the positive position the arm was made to vibrate ~~a little~~ by moving the index

extr	divis	time	point rest	time mut vib	
29.6					
21.1	—	— ~~25.2~~	25.2		
29.	—	— ~~25.17~~	25.17		
21.6					

Weights moved to negative position

extr	divis	time	point rest	time mut vib	
22.6					
	20	10.22.47	—	10.23.8	
	19	23.30			
16.3	—	—	19.27		
21.9	—	—	19.15		
16.5	—	—	19.1		
21.5	—	—	19.07		
16.8	—	—	19.07		
21.2	—	—	19.07		
17.1	—	—	19.05		
20.8	—	—	19.02		
17.4	—	—	19.05		
20.6	—	—	19.02		
	20	11.32.16	—	11.33.53	
	19	33.58			
17.5	—	—	18.97		
					7.13

extr	divis	time	point rest	time mid vib	
	19	11.41.16	— —	11.41.6	
	20	43.0			
20.3					

Weights moved to the positive position

extr	divis	time	point rest	time mid vib	
20.2					
	24	49.10	— —	11.49.37	
	26	50.19			
29.4			24.95		7.7
	26	56.15	— —	56.44	
	25	47			
20.8			24.92		
28.7			24.87		
21.3			24.85		
28.1			24.75		
21.5			24.67		
27.6			24.67		
22			24.7		
	24	0.45.48	— —	0.46.21	
	25	46.43			
27.2			24.7		7.1
	25	53.11	— —	53.22	
	24	54.9			
22.4					

Motion of arm on moving weights from pos to neg = 5.9
neg to pos = 5.98

Time one vib. when weights are in neg position = 7.5"
pos. position = 7.5

My reason for making the arm to vibrate at the beginning of the experiment was that as the weights were moved to the negative position immediately after the arm had arrived at the lowest extremity the arm had very little way to move before it came to the next extremity & consequently ~~that~~ it came to that extremity ~~was observed~~ very soon after the motion of the weights whereas in the preceding experiment it did not come to the next extremity till about ~~5 after the motion of the weights~~

In the 4th experiment the effect of the weights seemed to increase on standing in all 3 motions of the weights conformably to what was observed with the former wire but in the last experiment the case was different for though on moving the weights from positive to negative the effect seemed to increase on standing yet on moving them from negative to positive it diminished ~~on standing~~.

My next trials were to see whether this

effect was owing to magnetism Now, as it happend, the case in which the arm was inclosed was placed nearly parallel to the magnetic east & west & therefore if there was any thing magnetic in the balls & weights the balls would acquire polarity from the earth & the weights also after having remaind some time either in the positive or negative position would acquire polarity in the same direction & would attract the balls but when the weights were moved to the contrary position that pole which before pointed to the north would point to the south & would repel the ball it was approachd to but yet as repelling one ball towards the south has the same effect on the arm as attracting the other towards the north this would have no effect on the position of the arm After some time however the poles of the weight would be reversed & would begin to attract the balls

& would therefore produce the same kind of effect as was actually observed

To try whether this was the case I detatched the weights from the upper part of the copper rods by which they were suspended but still retaind the lower joint namely that which passed through them: I then fixed them in their positive position in such manner that they could turn round on this joint as a vertical axis I also made an apparatus by which I could turn them half way round on these vertical axes without opening the door of the room

Having sufferd the apparatus to remain in this manner for a day I next morning observed the arm & having found it to be stationary turned the weights half way round on their axes but could not perceive any motion in the arm Having sufferd ~~it to remain~~ the weights to produced in the place of the arm by turning them half round which not only confirms the deduction drawn from the former experiment but also seems to shew that in the experiments with the iron rods the effect produced could not be owing to magnetism.

The next thing which suggested it self to me was that possibly the effect might be owing to a difference of temperature between the weights & the case for it is evident that if the weights were much warmer than the case they would warm that side which was next to them & produce a current of air which would make the balls approach nearer to the weights Though I thought it not likely that there should be sufficient difference between the heat of the weights & case to have any sensible effect & though it seemed improbable ~~not likely~~ that in all the foregoing experiments the weights should

happen to be warmer than the case I resolved to examine into it & for this purpose removed the apparatus used in the last exper:
:iments & supported the weights by the copper rods as before & having placed them in the midway position I put a lamp under each & placed a thermometer with its ball close to the outside of the case near that part which one of the weights approached to in its ~~positive~~ positive position & in such manner that I could distinguish the divisions by the telescope Having done this I shut the door & some time after moved the weights to the positive position At first the arm was drawn aside only in its usual manner but in $\frac{1}{2}$ an hour the effect was so much increased that the arm was drawn 14 divisions aside instead of about 3 as it would otherwise have been & the thermometer was raised near $1\frac{1}{2}°$ namely from 61 to $62\frac{1}{2}$ On opening the door the weights were found to be no more

heated than just to prevent their feeling cool to my fingers

As the effect of a difference of temperature appeard to be so great I bored a small hole in one of the weights about $\frac{3}{4}$ of an inch deep & inserted the ball of a small thermometer & then coverd up the opening with cement Another small thermometer was placed with its ball close to the case & as near to that part to which the weight was approached as could be done with safety the ther:mometers being so placed that when the weights were in the negative position both could be seen through one of the telescopes by means of light reflected from a concave mirror

~~extreme~~ point	time	point of rest	~~Therm in~~ air	weight
Exp.	6	Sep. 6	weights in midway position	

Exp. 6 Sep. 6 weights in midway position

~~extr point~~	~~div~~	~~point rest~~	~~weight in air.~~	~~weight~~	
extr point	div	time	point rest	Therm in air	weight
	18.9	9.43	– –	55.5	
	18.85	10.3	18.85		
Weights		moved to	neg. posit		
13.1		10.12	– – – –	55.5	55.8
18.4	–	18	15.82		
13.4	–	.25			
missed		missed	– – –	55.5	55.8
13.6	– –	39			
17.6	–	46	15.65		
13.8	–	[illegible].53	15.65		
17.4	– –	11.0	15.65		
14.0	– –	.7	15.65		
17.2	– –	14	–	55.5	
Weights		moved to	pos.		
25.8		23			
17.5		30	21.55		
25.4		37	21.6		
18.1		44	21.65		
25.0		51			
missed					
24.7		0.5			
19.		12	21.77		
24.4		19			

Mot arm on moving weights from midway to – = 3.03
– to + – 5.9

Exp. 7 Sep. 18 weights in midway position

Time	div	extr.	point rest	therm in air	therm in weight
8.30	19.4	–	–	56.7	
9.32	19.4	–	–	56.6	
Weights moved to neg. position					
40	–	13.6		–	57.2
47	–	18.8	16.25		
54	–	13.8			
8 extreme points missed					
10.58	–	16.9			
11.5	–	14.5	15.62		
12	–	16.6			
Weights moved to pos.					
20	–	26.4		56.5	
28	–	17.2	21.72		
35	–	26.1			
4 extreme points missed					
0.10	–	19.3			
17	–	25.1	22.3		
24	–	19.7			

Mot. arm on moving weights from midway to – = 3.15
– to + = 6.1

Exp. 8 Sep. 23 Weights in midway position

Time	div	extr point	point of rest	therm in air	in weight
9.46	19.3	–	– –	53.1	
10.45	19.2	– –	19.2 –	53.1	
Weights moved to –					53.6
56	–	13.5	– –	–	
11. 3	–	18.6	16.07		
10	–	13.6			
4 extreme points missed					
44		17.4			
51		14.1	15.7		
58	–	17.2	– –	–	53.6
Weights moved to +					
0.1	–	15.7			
8	–	26.7	21.42		
15	–	16.6	– –	53.15	
2 extreme points missed					
36	–	25.9			
43	–	18.1	21.9		
50	–	25.5			

Mot arm on moving weights from midway to – = 3.13
– to + = 5.72

In ~~each of~~ these 3 experiments the effect of the weight appeard to increase ~~from~~ from 2 to 5 tenths of a division on standing an hour &

the thermometers shewed that the weights were 3 or 5 tenths of a degree warmer than the air close to the case In the 2 last experiments I put a lamp into the room over night in hopes of making the air warmer than the weights but without effect as the heat of the weights exceeded that of the air more in these 2 experi:ments than in the former

On the evening of oct 17 the weights being placed in the midway position lamps were put under them in order to warm them the door was then shut & the lamps sufferd to burn out The next morning ~~it~~ was found on moving the weights to the negative position that they were 7$\frac{1}{2}$° warmer than the air near the case After they had continued an hour in that position they were found to have coold 1$\frac{1}{2}$° so as to be only 6° warmer than the air They were then moved to the positive position & in both positions the arm was drawn aside about 4 divisions more after the weights had remaind an hour in that position than it was at first

May 22 1798 the exper. was repeated in the same manner except that the lamps were made so as to burn only a short time & only 2 hours were sufferd to elapse before the weights were moved The weights were now found be scarcely 2° warmer than the case & the arm was drawn aside about 2 divisions more after the weights had remaind an hour in the position they were moved to than ~~they were~~ it was at first

On May 23 the experiment was tried in the same manner except that the weights were coold by laying ice on them the ice being confined in its place by tin plates which on moving the weights fell to the ground so as not to be in the way On mo: :ving the weights to the negative position they were found to be about 8° colder than the air & their effect on the arm seemed

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now to diminish on standing instead of increasing as it did before as the arm was drawn aside about $2\frac{1}{2}$ divisions less at the end of an hour after the motion of the weights than it was at first

It seems sufficiently proved therefore that the effect in question is produced as above explaind by the difference of temperature between the weights & case for in the 6th 8th & 9th experiments in which the weights were not much warmer than the case their effect increased but little on standing whereas it increased much when they were much warmer than the case & decreased much when they were much cooler

It must be observed that in this apparatus the box in which the balls play is pretty deep & the balls hang near the bottom of it which makes the effect of the current of air more sensible than it would otherwise be & is a defect which I intend to rectify in some future experiments

Exp. 9 Apr. 29 weights in positive position

extreme point	divis	time	point rest	time of mid vib
34.7				
35	- -	- - -	34.84	
34.65				

Weights moved to —

extreme point	divis	time	point rest	time of mid vib
23.8				
	28	11.18.29	- - -	11.18.43
	29	58		
33.2	- -	- - -	28.52	
	29	25.27	- - - -	25.40
	28	- . 57		
23.9	- -	- -	28.25	
32	- -	- -	28.01	
24.15	-	- -	27.82	
31	-	- -	27.63	
24.4	-	- -	27.55	
30.4	-	- - -	27.47	
	28	0.7.4	- -	0.7.26
	27	53		
24.7	-	- -		

mot arm = 6.31
time vibs = 6.58

Exp. 10 May 5 weights in positive position

[..]adn	div	[..]ins	point rest	time mid vib	
34.5					
33.5	–	+	– – 33.97		
34.4					
Weights moved to –					
22.3		– –	–	43.36	
	28	43.42			
	29	44.6			7. 0
33.2	–	– –	27.82		
	28	50.33	– –	50.36	
	27	51.0			
22.6	–	– –	27.72		
32.5	–	– –	27.7		
23.2	–	– –	27.58	–	
31.45	–	– –	27.4		
23.5	–	– –	27.28		
	27	25.20	– –	25.24	
	28	58			
30.7	–	– –	27.21		7.3
	28	32.0	– –	32.27	
	27	32.40			
23.95	–	– –	27.21		6.56
	27	139.19	– –	39.23	
	28	140.2			
30.25					

mot. arm = 6.15

time vibr = 6:59

Exp 11 May 6. Weights in positive position

extr	div	time	point of rest	time mid div
34.9				
34.1	–	— –	34.47	
34.8	–	– –	34.49	
34.25				

Weight moved to –

extr	div	time	point of rest	time mid div
33.3				
	28	9.59.59	– – –	10.0.8
	29	10.0.27		
33.3		– – –	28.42	7.5
	29	6.52		
	27	7.51		
23.8	–	– – –	28.35	
32.5	–	– –	28.3	
24.4				

missed

extr	div	time	point of rest	time mid div
24.8				
31.3	–	– –	28.17	
	29	48.37	– –	49.8
	28	49.21		
25.3	–	–	28.2	
	28	56.8	– – –	56.13
	29	56.50		
30.9				

Mot arm = 6.18

time vibr = 7.0

In the 3 foregoing experiments the index was purposely moved so that before the beginning of the experiment the balls rested as near the sides of the case as they could without danger of touching it for it must be observed that when the arm is at 35 they begin to touch

In the 2 following experiments the index was in its usual position

Exp. 12 May 9 weights at −

extr	div	time	point rest	time mid
	17.4	9.45.0		
	17.4	53		
	17.4	10.8		
	17.[illegible]	10	17.4	

Weights moved to +

28.85				
	24	20.50		
	22	21.46	− − −	10.20.59
18.4	−	—	23.48	
28.3	−	—	23.57	
19.3	−	—	23.67	
27.8	−	− −	23.72	

extr	div	time	point rest	time mid
20	– –	– – –	23.8	
27.4	–	– – –	23.83	
	24	11.3.13	– – –	11.3.14
	23	54		
20.55	–	– – –	23.87	
	23	9.45	– – –	11.10.18
	24	10.28		
27				

Mot arm = 6.08
Time vibr = 7.3

Exp. 13 May 25 Weights at –

16				
18.3	– –	– – – –	17.2	
16.2				

Weights moved to +

29.6				
	25	22.22	– – –	22.56
	24	45		
17.4	–	– –	23.32	
	23	29.59	– – –	30.3
	24	30.23		
28.9	–	– –	23.4	
	24	36.58	– – –	37.7
	23	37.24		
18.4	–	– –	23.52	

extr	div	time	point rest	time mid
[illegible]	23	44.3	— — —	44.14
	24	31		
28.4	— —	— —	23.62	
19.3	—	— —	23.7	
27.8	—	— —	23.7	
	24	5.26	— — —	5.31
	23	6.1		
19.9	—	— —	23.72	
	23	12.12	— —	12.35
	24	50		
27.3				

Weights moved to —

extr	div	time	point rest	time mid
13.5				
21.8	—	— —	17.75	
	18	37.34	— —	37.39
	17	38.10		
13.9	—	— —	17.67	
	17	44.26	— —	44.45
	18	45.4		
21.1	—	— —	17.62	
14.4	—	— —	17.6	
20.5	—	— —	17.52	
14.7	—	— —	17.47	
20	—	— —	17.42	
	18	19.57	— —	20.24
	17	20.52		
15	—	— —	17.37	
	17	27.15	— —	27.30
	18	28.15		
19.5				

Mot arm on moving weights from − to + = ~~6.1??~~ 6.1250
+ to − = 8.97

Time vib. at + = 7.6
− = 7.9

Exp. 14 May 26 weights ~~in~~ − point

	16.1	9.18		
	16.1	24		
	16.1	46		
	16.1	49	16.1	

Weights moved to +

27.7				
	23	10.0.46	− −	10.1.1
	22	1.16		
17.3	−	− −	22.37	
	22	7.58	− − −	8.5
	23	8.27		
27.2	−	−	22.5	
	23	15.2	− − −	15.9
	22	32		
18.3	−	− −	22.65	
26.8	−	−	22.75	
19.1	−	− −	22.85	
26.4	−	− −	22.97	
	23	43.40	− −	43.32
	22	44.22		
20	−	− −	23.15	
	22	49.53	− −	50.41
	23	50.37		
26.2				

Weights. moved to —

12.4				
	16	11.7.53	— — —	8.25
	17	8.27		
21.5	—	— —	17.02	
	17	15.30	— — —	15.27
	16	16.3		
12.7	—	— —	16.9	
20.7	—	— —	16.85	
13.3	—	— —	16.82	
20	—	— —	16.72	
13.6	—	— —	16.67	
	16	50.33	— — —	50.58
	17	51.19		
19.5	—	—	16.65	58.6
	17	57.53		
	16	58.44		
14				

Mot arm by moving weights from − to + = 6.27
+ to − = 6.13

Time vib at + = 7.6
− = 7.6

In the next exper the balls before the motion of the weights were made to rest as near as possible to the sides of the case ~~as I could~~ but on the contrary side from what they did in the 9. 10 & 11th exper

Exp. 15 May 27 weights in neg position

3.9				
3.35	—	— —	3.61	
3.85	—	— —	3.61	
3.4				
	Weights moved			to +
15.4				
	10	10.5.59	— — —	10.5.56
	9	6.27		
4.8	—	— — —	9.95	
	9	12.43	— — — —	13.5
	10	13.11		
14.8	—	— — —	10.07	
	10	20.24	— — —	20.13
	9	56		
5.9	—	— — —	10.23	
14.35	—	— —	10.35	
6.8	—	— —	10.46	
13.9	—	— — —	10.52	
	11	48.30	— —	48.42
	10	49.11		
7.5	—	— —	10.6	
	10	55.26	— — —	55.48
	11	56.10		
13.5				

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mot arm — — 6.34

time vibr — — 7.7

The 2 following experiments were made by Mr Gilpin who was so good as to assist me on the occasion

Exp. 16 May 29 weights in neg position

22.55				
8.4	— —	~~15.08~~	15.08	
21	———	~~14.9~~	14.9	
9.2				

Weights moved to +

26.6				
	22	10.22.53	10.22.53	10.23.15
	21	23.20	23.20	
15.8	—	—	21	
	20	30.7	— —	30.30
	21	36		
25.8	—	—	21.05	
	22	37.23	— —	37.45
	21	55		
16.8	—	—	21.11	
	20	44.29	— —	45.1
	21	45.4		
25.05	—	—	21.11	
	22	51.54	— —	52.20
	21	52.32		
17.57	—	—	21.2	
	20	59.31	— —	59.34
	22	11.0.13		
24.6	—	—	21.28	

	22	11.6.24	— —	11.6.49
	21	7.9		
18.3				

mod arm = 6.1

time vibt = 7.16

Exp. 17 May 30 weights at —

17.2	10.19
17.1	25
17.07	29
17.15	40
17.45	49
17.42	51
17.42	11.1

Weights moved to +

28.8				
	24	11.11.23	— —	11.11.37
	23	49		
18.1	—	— —	23.2	
	22	18.13	— —	18.42
	23	43		
27.8	—	— —	23.12	
	24	25.19	— —	25.40
	23	49		
18.8	—	— -	23.2	
	23	32.41		32.43
	24	33.13		
27.38	—	— — .	23.31	

	24	39.28	—	39.44
	23	40.3		
19.7			23.44	
	23	46.33	— —	46.46
	24	47.11		
27.			23.52	
	24	53.36	— —	53.48
	23	54.17		
20.4			23.57	
	23	0.0.34	— —	0.0.55
	24	1.18		
26.5			23.55	
	24	7.34	— —	7.50
	23	8.21		
20.8			23.53	
	23	14.30	— —	14.58
	24	15.24		
26.25				

Weights moved to —

13.3				
	17	32.19	— —	32.44
	18	48		
22.4			17.95	
	18	39.46	—	39.44
	17	40.19		
13.7			17.85	
	17	46.26	— —	46.48
	18	47.0		
21.6			17.72	
	18	53.43	— —	53.20
	17	54.20		

14.	–	–	17.6	
	17	0.0.39	– –	0.0.55
	18	1.20		
20.8	—		17.47	
	19	7.39	–	7.59
	17	8.21		
143	–	–	17.37	
	17	14.54	– –	15.4
	18	15.42		
20.1	—	–	17.27	
	18	·21.32	— –	22.5
	17	22.22		
14.6				

Mot arm on moving weight from – to + = 5.78
+ to – 5.63
time vibr at + = 7
– = 7.

On the method of computing the density of the earth from these experiments

I shall first compute this on the supposition that the arm & copper rods have no weight & that the weights exert no sensible attraction except on the nearest ball & shall then examine what corrections are necessary on account of the arm & rods & some other small causes

The first thing is to find the force required to draw the arm aside which as was before said is to be determined by the time of a vibration

The distance of the centers of the 2 balls from each other is 73,3 inches & therefore the distance of each from the center of motion is 36,65 & the length of a pendulum vibrating seconds in this climate is 39,14. therefore if the stiffness of the wire by which the arm is suspended is such that the force ~~required~~ which must be applied to each ball in order to draw the arm aside by the angle A is to the weight of that ball as the arch of A to the radius. the arm will vibrate in the same time as a pendulum whose length is 36.65 inches that is in $\sqrt{\frac{36.65}{39.14}}$ seconds & therefore if the stiffness of the wire is such as to make it vibrate in N seconds the force which must be applied to each ball in order to draw it aside by the angle A is to the weight of the ball as the arch of A $\times \frac{1}{N^2} \times \sqrt{\frac{36.65}{39.14}}$ to the radius But the ivory scale at the end of the arm is 38.3 inches from the center of motion & each division is $\frac{1}{20}$ of an inch & therefore subtends an angle at the center whose arch is $\frac{1}{766}$ & therefore the force which must be applied to each ball to draw the arm aside by one division is to the weight of the ball as $\frac{1}{766 N^2} \sqrt{\frac{36.85}{39.14}}$ to one or as $\frac{1}{818 N^2}$ to one

The next thing is to find the proportion which the attraction of the weight on the ball bears to that of the ~~attraction of the~~ earth thereon ~~that is to its weight When the weights are approachd to the balls~~. supposing the ball to be placed in the middle of the case that is to be not nearer to one side than the other When the weights are approachd to the balls their centers are 8.85 inches from the middle line of the case ~~which~~ but through ~~inad~~ inadvertence the distance from each other of the rods which support these weights ~~from each other~~ was made equal to the distance of the centers of the balls from each other whereas it ought to have been somewhat greater In consequence ~~of which~~ they ~~the centers of the weight when the~~ weights are approachd to the balls their centers do not stand exactly opposite to each other & the effect ~~of the weights~~ in drawing ~~the arm aside is less than it would otherwise~~ ~~In~~ of this the centers of the weights are not exactly opposite to those of the balls when they are approached together & the effect of the weights in drawing the arm aside is less than it would otherwise have been in the triplicate ratio of $\frac{8.85}{36.65}$ to the chord of the angle whose sine is $\frac{8.85}{36.65}$ or in the triplicate ratio of the cosine of $\frac{1}{2}$ this angle to the radius or in the ratio of .9779 to one

Each of the weights weighs 2439000 grains & therefore is equal in weight to 10.64 spherical feet of water & therefore its attraction on a particle placed at the center of the ball is to that attraction of a spherical foot of water on an equal particle placed on its surface as $10.64 \times .9779 \times \left.\frac{6}{8.85}\right]^2$ to one The mean diameter of the earth is 41800000 feet *

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* In strictness we ought instead of the mean diameter of the earth to take the diameter of that sphere whose attraction is equal to the force of gravity in this climate but the difference is not worth regarding

& therefore if the mean density of the earth is to that of water as Δ to one the attraction of the leaden weight on the ball will be to that of the earth thereon as $10{,}64 \times .9779 \times \overline{\frac{6}{8{,}85}}\Big|^2$ to $41800000\,\Delta$ :: 1 to $8739000.\,\Delta$

It is shewn therefore that the force which must be applied to each ball in order to draw the arm one division out of its natural position is $\frac{1}{818\,N^2}$ of the weight of the ball & if the mean density of the earth is to that of water as Δ to 1 the attraction of the weight on the ball is $\frac{1}{8739000\,\Delta}$ of the weight of that ball & therefore the attraction will be able to draw the arm out of its natural position by $\frac{818\,N^2}{8739000\,\Delta}$ or $\frac{N^2}{10683\,\Delta}$ divisions & therefore if on moving the weights from the midway to a near position the arm is found to move B divisions or if it moves 2B divisions on moving the weights from one near position to the other it follows that the density of the earth or D is $\frac{N^2}{10683\ B}$

We must now consider the corrections which must be applied to this result first for the effect which the resistance of the arm to motion has on the time of the vibration 2nd for the attraction of the weights on the arm 3rd for their attraction on the further ball 4th for the attraction of the copper rods on the balls & arm 5th for the attraction of the case on the balls & arm & 6th for the alteration of the attraction of the weights on the balls according to the position of the arm & the effect which that has on the time of vibration None of these corrections indeed except the last are of much signification but they ought not intirely to be neglected

As to the first it must be considerd that during the vibrations of the arm & balls part of the force is spent in accelerating the arm & therefore in order to find the force required to draw them out of their natural position we must find the proportion which the forces spent in accelerating the arm & balls bear to each other

Let EDCedc fig 4 be the arm B & b the balls Cs the suspending wire The arm consists of 4 parts first a deal rod Dd 73.7 inches long 2nd the silver wire Dd weighing 170 grn 3rd the end pieces DE & Ed to which the ivory vernier is fastend each of which weighs 45 grn & 4th some brass work Cc at the center The deal rod when dry weighs 2320 grn but when very damp as it commonly was during the experiments weighs 2400 the transverse section is of the shape represented in fig 5 the thickness Bd & the dimensions of the part DEed being the same in all parts but the breadth Bb diminishes gradually from the middle to the ends The area of this section is .33 of a square inch at

the middle & ,146 at the end & therefore if any point x, (fig 4) is taken in cd & $\frac{cx}{cd}$ is called x this rod weighs $\frac{2400 \times .33}{73.3 \times .238}$ per inch at the middle $\frac{2400 \times .146}{73.3 \times .238}$ at the end & $\frac{2400}{73.3} \times \frac{.33 - .184x}{.238} =$
$= \frac{3320 - 1848x}{73.3}$ at x & therefore as the weight of the wire is $\frac{170}{73.3}$ per inch the deal rod & wire together may be considered as a rod whose weight at $x = \frac{3490 - 1848x}{73.3}$ per inch

But the force required to accelerate any quantity of matter placed at x is proportional to x^2 that is it is to the force required to accelerate the same quantity of matter placed at d as x^2 to 1 & therefore if cd is called l & x is supposed to flow the fluxion of the force required to accelerate the deal rod & ~~~~ wire is proportional to $\frac{x^2 l \dot{x} \times \overline{3490 - 1848x}}{73.3}$ the fluent of which generated while x flows from c to d $= \frac{l}{73.3} \times \frac{3490}{3} - \frac{1848}{4} = 350$ so that the force required to accelerate each half of the deal rod & wire is the same as is required to accelerate 350 grs. placed at d

The resistance to motion of each of the pieces de is equal to that of 48 grs placed at d as the distance of their centers of gravity from C is 38 inches that of the brass work at the center may be disregarded & therefore the whole force required to accelerate the arm is the same as that required to accelerate 398 grs placed at each of the points D & d

Each of the balls weigh 11262 grs & are placed at the same distance from the center as D & d & therefore the force required to accelerate the balls & arm together is the same as if each ball weighed 11661 & the arm had no weight & therefore ~~the force required to draw the arm aside~~ supposing the time of a vibration to be given the force required to draw the arm aside is greater than if the arm had no weight in the proportion of 11660 to 11262 or of 1.0353 to 1

To find ~~On~~ the attraction of the weights on the arm & ~~the particles [illegible]~~ [illegible]

through d draw the vertical plane dwb per:
:pendicular to Dd & let w be the center of the weight which though not accurately in this plane may without sensible error be considered as placed therein let b be the center of the ball then wb is horizontal & $= 8.85$ & db is vertical & $= 5.5$ let $wa = a$ $wb = b$ & let $\frac{dx}{dc}$ or $1 - x = z$ then the attraction of the weight on ~~the point~~ a particle of matter at x in the direction dw is to its attraction on the same particle placed at $b :: b^3 : \overline{a^2 + z^2 l^2}^{\frac{3}{2}}$ or is proportional to $\frac{b^3}{\overline{a^2+z^2 l^2}^{\frac{3}{2}}}$ & the force of that attraction to move the arm is proportional to $\frac{b^3 \times \overline{1-z}}{\overline{a^2+z^2 l^2}^{\frac{3}{2}}}$ & the weight of the deal rod & wire at the point x was before said to be $\frac{3490 - 1848\,x}{73.3} = \frac{1642 + 1848\,z}{73.3}$ per inch & therefore if $\dot{x}$ flows the fluxion of the power to move the arm

$$= l\dot{z} \times \frac{1642 + 1848\,z}{73.3} \times \frac{b^3 \times \overline{1-z}}{\overline{a^2 + z^2 l^2}^{3/2}}$$

$$= \dot{z} \times \overline{821 + 924z} \times \frac{b^3 \times \overline{1-z}}{\overline{a^2+l^2z^2}|^{\frac{3}{2}}} = \frac{b^3\dot{z} \times \overline{821 + \overset{64}{103}z - 924z^2}}{\overline{a^2+l^2z^2}|^{\frac{3}{2}}} =$$

$$= \frac{b^3\dot{z} \times \overline{821 + 103z + \frac{924a^2}{l^2}}}{\overline{a^2+l^2z^2}|^{\frac{3}{2}}} - \frac{924\, b^3\dot{z} \times \overline{\frac{a^2}{l^2} + z^2}}{\overline{a^2+l^2z^2}|^{\frac{3}{2}}}$$ which

as $\frac{a^2}{l^2} = .08$ $= \frac{b^3\dot{z} \times \overline{895 + 103z}}{\overline{a^2+l^2z^2}|^{\frac{3}{2}}} - \frac{924\, b^3\dot{z}}{l^2\sqrt{a^2+l^2z^2}}$ The

fluent of this $= \frac{895\, b^3 z}{a^2\sqrt{a^2+l^2z^2}} - \frac{103\, b^3}{l^2\sqrt{a^2+l^2z^2}} + \frac{103\, b^3}{l^2 a}$

$- \frac{924\, b^3}{l^3}\, L.\frac{lz + \sqrt{a^2+l^2z^2}}{a}$ & the force with which the attraction of the weight on the nearest half of the deal rod & wire tends to move the arm is proportional to this fluent generated while z flows from 0 to 1 that is to 128 grs.

The force with which the attraction of the weight on the end piece de tends to move the arm is propor:tional to $47 \times \frac{b^3}{a^3}$ or 29 grs & therefore the whole power of the weight to move the arm by means of its attraction on the nearest part thereof is equal to its attraction on 157 grs. placed at b which is

$\frac{157}{11260}$ or .0139 of its attraction on the ball

It must be observed that the effect of the attraction of the weight on the whole arm is rather less than this as its attraction on the farther half draws it the contrary way but as the attraction on this is small in comparison of its attraction on the nearer half it may be disregarded

The attraction of the weight on the furthest ball in the direction ~~da~~ bw is to its attraction on the nearest ball :: $wd^3 : wD^3$ :: .0017 : 1 & therefore the effect of the attraction of the weight on both balls is to that of its attraction on the nearest ball :: .9983 : 1

To find the attraction of the copper rod on the nearest ball let b & w (19) 6 be the centers of the ball & weight & ea the perpendicular part of the copper rod which consists of 2 parts ad & de ad weighs 22000 grs & is 16 inches long & is nearly bisected by w de weighs 41000 & is 46 inches long wb is 8,85 attraction of the weight & copper rod on the arm & both balls together exceeds the attraction of the weight on the nearest ball in the proportion of ·9983 + ·0139 + ·0077 to one or of 1,0199 to 1 inches & is perpendicular to en Now the attraction of a line an of uniform thickness on b in the direction bn is to that of the same quantity of matter placed at n :: $bn : eb$ & therefore the attraction of the part da equals that of $\frac{22000 \times nb}{db}$ or 16300 placed at n & the attraction of de equals that of $41000 \times \frac{en}{ed} \times \frac{bn}{be} - 41000 \times \frac{dn}{ed} \times \frac{bn}{bd}$ or 2500 placed at the same point so that the attraction of the perpendicular part of the copper rod on b is to that of the weight thereon as 18800 : 2439000 or as ,00771 to 1 as for the attraction of the inclined part of the rod & wooden bar marked Pr & rr in fig 1 it may safely be neglected & so may the attraction of the whole rod on the arm & farthest ball & therefore the

The next thing to be considered is the attraction of the mahogany case Now it is evident that when the arm stands at the middle division the attractions of the opposite sides of the case balance each other & have no power to draw the arm either way When the arm is removed from this division it is attracted a little towards the nearest side so that the force required to draw the arm aside is rather less than it would otherwise be but yet if this force is proportional to the distance of the arm from the middle division it makes no error in the result for though the attraction will draw the arm aside more than it would otherwise do yet as the accelerating force by which the arm is made to vibrate is diminished in the same proportion the square of the time of a vibration will be increased in the same proportion as the space by which the arm is drawn aside & therefore the result will be the same as if the case exerted no attraction but if the attraction of the case is not proportional to the distance of the arm from the middle point the ratio in which the accelerating force is diminished is different in different parts of the vibration & the square of the time of a vibration will not be increased in the same proportion as the quantity by which the arm is drawn aside & therefore the result will be alterd thereby

On computation I find that the force by which the attraction draws the arm from the center is far from being proportional to the distance but the whole force is so small as not to be worth regarding for in no position of the arm does the attrac: :tion of the case on the balls exceed that of $\frac{1}{5}$ of a spheric inch of water placed at the distance of 1 inch from the center of the balls & the attraction of the leaden weight equals that of 10,6 spheric feet of water placed at 8,85 inches or of 234 spheric inches placed at 1 inch distance so that the attraction of the case on the balls can in no position of the arm exceed $\frac{1}{1170}$ of that of the weight. The computation is given in the appendix

It has been shewn therefore that the force required to draw the arm aside one division is greater than it would be if the arm had no weight in the ratio of 1,0353 to 1 & therefore $= \frac{1{,}0353}{818{,}0\, N^2}$ of the weight of the ball & moreover the attraction of the weight & copper rod on the arm & both balls together exceeds the attraction of the weight on the nearest ball in the ratio of 1,0197 to 1 & therefore $= \frac{1{,}0199}{8739000\, D}$ of the weight of the ball. Consequently D is really equal to $\frac{818{,}0\, N^2}{1{,}0353} \times \frac{1{,}0199}{8739000\, B}$ or $\frac{N^2}{10844\, B}$ instead of $\frac{N^2}{1{,}0683\, B}$ as by the former computation.

It remains to be considered how much this is affected by the position of the arm

Suppose the weights to be approached to the balls let W fig 7 be the center of one of the weights let M be the center of the nearest ball at its mean position or when the arm is at 20 division let B be the point which it actually rests at & let A be the point which it would rest at if the weight was removed consequently AB is the space by which it ~~is drawn aside~~ ~~would~~ is drawn aside by means of the attraction & let Mβ be the space by which it would be drawn aside if the attraction on it was the same as when it is at M But the attraction at B is greater than at M in the proportion of $WM^2 : WB^2$ & therefore $AB = M\beta \times \frac{WM^2}{WB^2} = M\beta \times 1 + \frac{2MB}{MW}$ very nearly

Let now the weights be moved to the contrary near position & let w be now the center of the nearest weight & b the point of rest of the center of the ball then Ab =

$= M\beta \times 1 + \frac{2Mb}{MW}$ & $Bb = M\beta \times 2 + \frac{2Mb}{MW} + \frac{2MB}{MW} =$
$= 2M\beta \times 1 + \frac{Bb}{MW}$ so that the whole motion Bb
is greater than it would be if the attraction on
the ball was the same in all places as it is
at M in the ratio of $1 + \frac{Bb}{MW}$ to one & therefore
does not depend sensibly on the place of the
arm in either position of the weights but
only on the quantity of its motion by moving
them

This variation in the attraction of the weight
affects also the time of vibration for suppose
the weights to be approached to the balls let
W be the center of the nearest weight let B α
A represent the same things as before & let x
be the center of the ball at any point of its
vibration let AB represent the force with
which the ball when placed at B is drawn
towards A by the stiffness of the wire then
as B is the point of rest the attraction of
the weight thereon will also equal AB &

when the ball is at x the force with which it is drawn towards A by the stiffness of the wire $= Ax$ & that with which it is drawn in the contrary direction by the attraction $= AB \times \frac{WB^2}{Wx^2}$ so that the actual force by which it is drawn towards $A = Ax - \frac{AB \times WB^2}{Wx^2} =$ $AB + Bx - AB \times 1 + \frac{2Bx}{WB} = Bx - \frac{2Bx \times AB}{WB}$ very nearly so that the actual force with which the ball is drawn towards the middle point of the vibration is less than it would be if the weights were removed in the ratio of $1 - \frac{2AB}{WB}$ to one & the square of the time of a vibration is increased in the ratio of 1 to $1 - \frac{2AB}{WB}$ ~~or of~~ $\cancel{1 + \frac{Bb}{WM}}$ ~~very nearly which~~ which differs very little from that of $1 + \frac{Bb}{MW}$ to 1 which is the ratio in which the motion of the arm by moving the weights from one near position to the other is increased the motion of the ball answering to one division of the arm $= \frac{36.85}{20 \times 38.5}$ & if mB is the motion of the ball answering to d divisions on the arm $\cancel{\frac{mB}{WM}} = \frac{MB}{WM} = \frac{36.85\,d}{20 \times 38.5 \times 8.85} =$ $= \frac{d}{185}$ & therefore the time of vibration & motion of the arm must be corrected as follows

If the time of vibration is determined by an experiment in which the weights are in the near position & the motion of the arm by moving the weights from the near to the midway position is d divisions the observed time must be diminished in the subduplicate ratio of $1 - \frac{2d}{185}$ to one that is in the ratio of $1 - \frac{d}{185}$ to one but when it is determined by an experiment in which the weights are in the midway position no correction must be applied

To correct the motion of the arm caused by moving the weights from ~~the near~~

~~position~~ a near to the midway position or the reverse observe how much the position of the arm differs from 20 divisions when the weights are in the near position let this be n divisions then if the arm at that time is on the same side of the division of 20 as the weight the observed motion must be diminished by the $\frac{2n}{185}$ part of the whole but otherwise it must be as much increased

If the weights are moved from one near position to the other & the motion of the arm is $2d$ divisions the observed motion must be diminished by the $\frac{2d}{185}$ part of the whole

If the weights are moved from one near position to the other & the time of vibration is determined while the weights are in one of those positions there is no need of correcting either the motion of the arm or the time of vibration

Conclusion

The following table contains the result of the experiments

exp	Mot. weight	Mot arm	D° corr	Time vib	D° corr	D	D
1	m to +	14.32	13.42	1 — "	— —	5.64	5.53
	+ to m	14.1	13.17	14.53	— —	5.76	5.63
2	m to +	15.87	14.69	—	—	5.26	5.17
	+ to m	15.45	14.14	14.42 ~~15.4~~	— —	6.28	5.38
3	+ to m	15.22	13.56	14.39	— —	5.38	5.30
	m to +	14.5	13.28	14.53	— —	5.69	5.58
4	m to +	3.1	2.95	— — —	6.54	5.49	5.40
	+ to −	6.18	— —	7.1	— —	5.42	5.33
	− to +	5.92	— —	7.3	— —	5.71	6.62
5.	+ to −	5.9	— —	7.5	— —	5.79	5.69
	− to +	5.98	— —	7.5	— —	5.68	5.75
6	m to −	2.98	2.85	— —	— —	5.66	5.67
	− to +	5.9	5.71	— —	— —	5.87	5.48
7	m to +	3.15	3.03	— — —	6.56	5.41	5.34
	− to +	6.17	5.9	— —	— —	5.54	5.30
8	m to −	3.13	3.00	— —	— —	5.46	5.39
	− to +	5.72	5.54	—	— —	5.92	5.65
9	+ to −	6.32	— —	6.58	— —	5.24	5.14
10	+ to −	6.15	— —	6.59	— —	5.38	5.31
11	+ to −	6.07	— —	7.1	— —	5.40	5.43
12	− to +	6.09	— —	7.3	— —	5.54	5.46
13	− to +	6.12	— —	7.6	— —	5.58	5.51
	+ to −	5.97	— —	7.7	— —	5.80	5.68
14	− to +	6.27	— —	7.6	— —	5.66	5.38
	+ to −	6.13	— —	7.6	— —	5.64	5.50

15	- to +	6.34	- -	7.7	- -	~~5.44~~	5.35
16	- to +	6.1	- -	7.16	- -	~~5.89~~	5.80
17	- to +	5.78	- -	7.2	-	- -	5.73
	+ to -	5.64	- -	7.3	-	-	5.90

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From this table it appears that though th experiments agree pretty well together yet t difference between them both in the quantity of motion of the arm & in the time of vibration is greater than can proceed merely from th error of observation as to the difference in the motion of the arm it may very well be accounted for from the current of air produced by the difference of temperature but whether this can account for the difference in the time of vibration is doubtful If the current of air was regular & of the same swiftness in all parts of the vibration of the ball I think it could not but as there will most likely be much irregularity in the current it may very likely be sufficien to account for ~~it~~ the difference

By a mean of the experiments made with the wire first used the density of the earth comes out 5.43 times greater than that of water & by a mean of those made with the latter wire it comes out 5.5~~0~~ & the extreme difference of the results of the 21 observations made with this wire is only .76 so that the extreme results do not differ from the mean by more than .38 or $\frac{1}{14}$ of the whole & therefore the density should seem to be determined hereby to great exactness It indeed may be objected that as the result appears to be influenced by the current of air or some other cause the laws of which we are not well acquainted with this cause may perhaps act always or commonly in the same direction & thereby make a considerable error in the result But yet as the experiments were ~~made~~ tried in various weathers & with considerable variety in the difference of temperature of the weights & air & with the arm resting at different distances from the sides of the case it seems very unlikely that this cause should act so uniformly in the same way as to make the error of the mean result near equal to the difference between this & the ~~mean result~~ extreme & therefore it seems very unlikely that the density of the earth should differ from 5.59 by so much as $\frac{1}{10}$ of the whole

Another objection perhaps may be made to these experiments namely that it is uncertain whether in these small distances the force of gravity follows exactly the same law as in greater distances There is no reason however to think that any irregularity of this kind takes place until the bodies come within the action of what is called the attraction of cohesion & which seems to extend only to very minute distances With a view to see whether the result could be affected by this attraction I made the 9th. 10th. 11th & 15th exper in which the balls were made to rest as close to the sides of the case as I could but there is no difference to be depended on between the results under that circumstance & when the balls are placed in any other part of the case

According to the experiments made by Dr Maskelyne on the attraction of the hill Shehallien the density of the earth is 4 $\frac{1}{2}$ times that of water which differs rather more from the preceding determination than I should have expected But I forbear entering into any consideration of which determination is most to be depended on till I have examined more carefully how much the preceding determination is affected by ~~causes~~ irregularities ~~whose quantity~~ I cannot measure

Appendix

On the attraction of the mahogany case on the balls

The first thing is to find the attraction of the rectangular plane $ck\beta b$ (fig 8) on the point a placed in the line ac perpendicular to this plane

Let $ac = a$ $ck = b$ $cb = x$ & let $\frac{a^2}{a^2+x^2} = w^2$ & $\frac{b^2}{a^2+w^2} = v^2$ then the attraction of the line $b\beta$ on a in the direction $ab = \frac{b\beta}{ab \times a\beta}$ & therefore if cb flows the fluxion of the attraction of the plane on the point a in the direction

$$cb = \frac{b\dot{x}}{\sqrt{a^2+x^2} \times \sqrt{a^2+b^2+x^2}} \times \frac{x}{\sqrt{a^2+x^2}} = \frac{-b\dot{w}}{w\sqrt{b^2+\frac{a^2}{w^2}}}$$

$$= \frac{-b\dot{w}}{\sqrt{b^2w^2+a^2}} = \frac{-\dot{v}}{\sqrt{1+v^2}}$$ the variable part of the fluent of which $= -\text{Log.}\ v+\sqrt{1+v^2}$ & therefore

the whole attraction $= \text{Log}\ \overline{\dfrac{ck+ak}{ac} \times \dfrac{ab}{b\beta+a\beta}}$.

that the attraction of the plane in the direction cb is found readily by logarithms but I know no way of finding its attraction in the direction ac except by an infinite series

The 2 most convenient series I know are the following

1st series let $\frac{b}{a} = \pi$ & let $A =$ arc whose tan is π $B = A - \pi$ $C = B + \frac{\pi^3}{3}$ $D = C - \frac{\pi^5}{5}$ &c

then the attraction in the direction ac =

$$\sqrt{1-v^2} \times A + \frac{Bv^2}{2} + \frac{3Cv^4}{2 \cdot 4} + \frac{3 \cdot 5 v^6}{2 \cdot 4 \cdot 6} \text{ \&c}$$

For the 2nd series let $A =$ arc whose tan $= \frac{1}{\pi}$ $B = A - \frac{1}{\pi}$ $C = B + \frac{1}{3\pi^3}$ $D = C - \frac{1}{5\pi^5}$ &c then

the attraction $=$ ar. $90^\circ - \overline{\sqrt{1+v^2} \times A - \frac{Bv^2}{2} + \frac{3Cv^4}{2 \cdot 4} - \frac{3 \cdot 5 D v^6}{2 \cdot 4 \cdot 6} \text{ \&c}}$.

It must be observed that the first series fails when π is greater than unity & the 2nd when it is less but if b is taken equal to the least of the 2 lines ck & cb there is no case in which one or the other of them may not be used conveniently

By the help of these series I computed the following table

	.1962	.3714	.5145	.6248	.7071	.7808	.8575	.9285	.9815	1.
.1962	.00001									
.3714	.00039	00148								
.5145	.00074	00277	00521	~~01182~~						
.6248	00110	00406	00778	01183						
.7071	00140	00522	01008	01525	02002					
.7808	00171	00637	01245	01896	02405	03247				
.8575	00207	00772	01522	02339	03116	03964	05057			
.9285	00244	00910	01810	02807	03778	04867	06319	08119		
.9815	00271	01019	02084	03193	04368	05639	07478	09931	12849	
1.	00284	01054	02135	03347	04560	05975	07978	10789	14632	19612

Find in this table with the argument $\frac{ck}{ak}$ at top & the argument $\frac{ck}{ak}$ in the left hand column the corresponding logarithm then add together this logarithm the logarithm of $\frac{ck}{ak}$ & the logarithm of $\frac{cb}{ab}$ The sum is logarithm of the attraction

To compute from hence the attraction of the case on the ball let the box DCBA (fig. 1) in which the ball plays be divided into 2 parts by a vertical section ~~passing through the section center of the ball~~ perpendicular to the length of the case & passing through the center of the ball & in fig 9 let the parallelopiped ABDGabde be one of these parts ABDG being the above mention vertical section let x be the center of the ball & draw the parallelogram βnpmδx parallel to BbdD & xqrp parallel to βBbn & bisect βδ in c: ~~then if we take~~ Now the dimensions of the box ~~such as they are~~ on the inside are

Bb = 1.75 BD = 3.6 Bβ = 1.75 & βA = 5

whence I find that if xc & βx are taken as in the 2 upper lines of the following table the attractions of the different parts are as set down below

xc — — — — —	.75	.5	.25
βx — — — — —	1.05	1.3	1.55
excess attr. Ddrg above Bbrg	.2374	.1614	.0813
— — mdrp above nbrp —	.2374	.1614	.0813
— — mesp above nasp	.3705	.2516	.1271
sum of these — —	.8453	.5744	.2897
excess attr Bbnβ above Ddmδ —	.5007	.3271	.1606
— — Aanβ above Eemδ —	.4677	.3079	.1525
Whole attraction of the inside surface of the half box —	.1231	.0606	.0234

It appears therefore that the attraction of the box on x increases faster than in proportion to the distance xc

The specific gravity of the wood used in this case is .61 & its thickness is $\frac{3}{4}$ of an inch & therefore if the attraction of the outside surface of the box was the same as that of the inside the whole attraction of the box on the ball when cx = .75 would be equal to $2 \times .1231 \times .61 \times \frac{3}{4}$ cubic inches or .201 spheric inches of water placed at the distance of 1 inch from the center of the ball & in reality it can never be so great as this as the attraction of the outside surface is rather less than that of the inside & moreover the distance of x from c can never be quite so great as .75 of an inch as the greatest motion of the arm is only $1\frac{1}{2}$ inch

~~may not be used conveniently~~

Fig 1

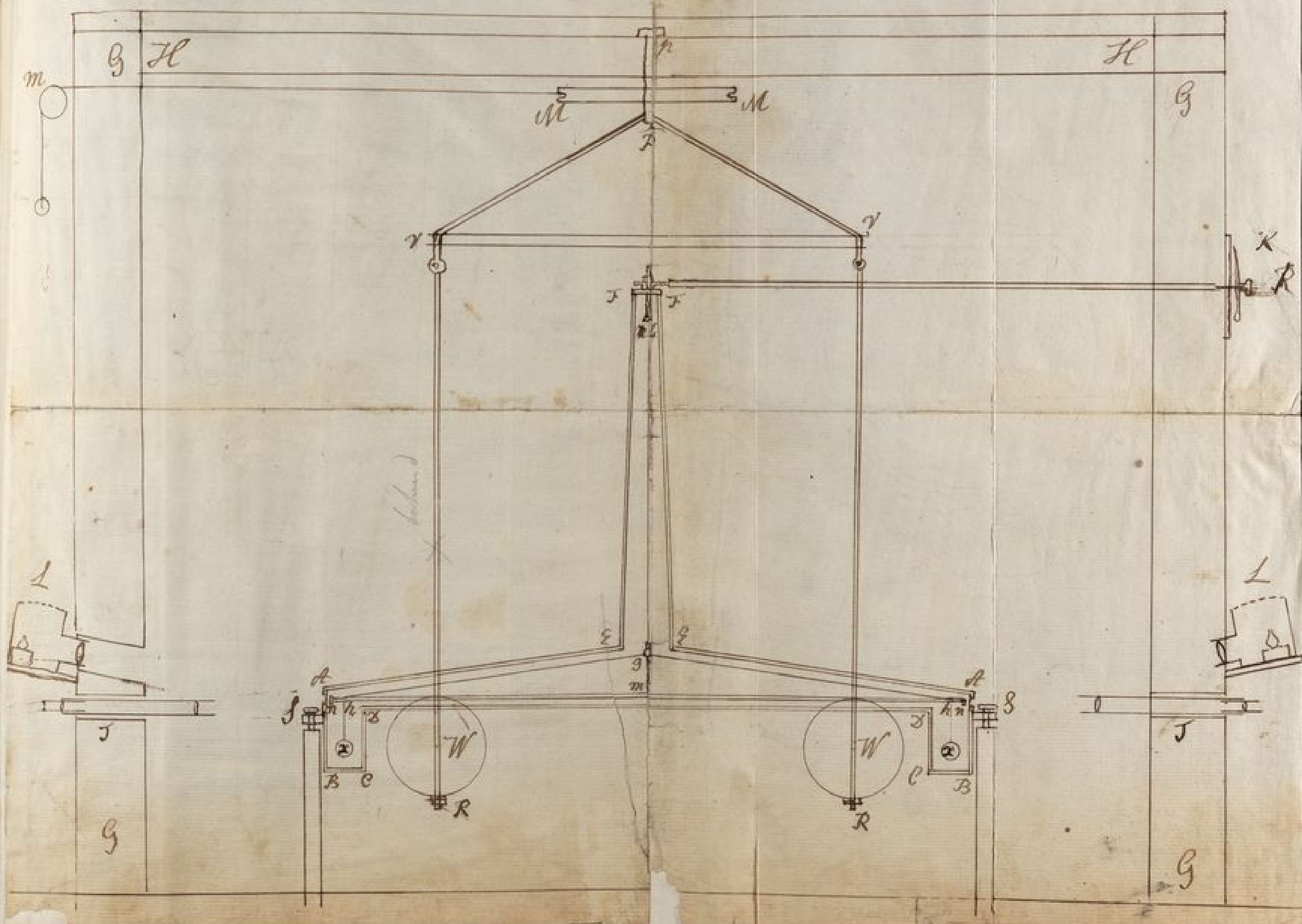

Fig 8

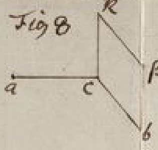

Fig 9

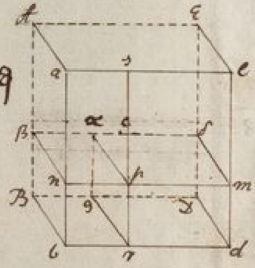

Fig 2

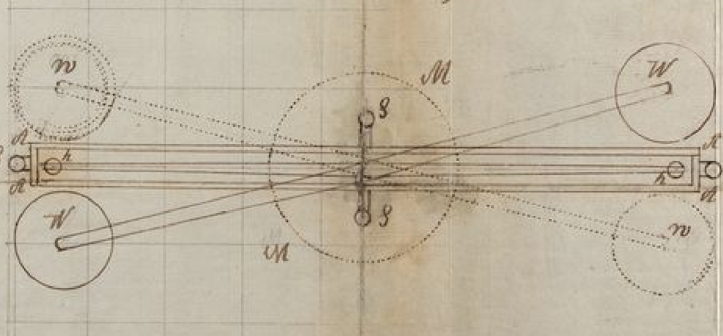

Fig. 3

Fig 7

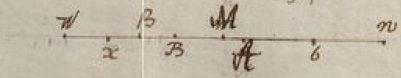

Fig. 5

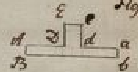

Fig 4

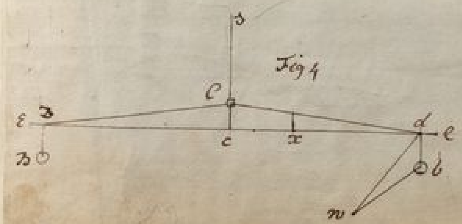

Fig 6